Maximum likelihood estimation for the seasonal Neyman-Scott rectangular pulses model for rainfall

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ABSTRACT

Simulation of rainfall records is usually performed in hydrology by applying cluster models, which have been proved to be able to satisfactorily fit the main statistics of rainfall data observed on a wide range of time scales. Among these, the Neyman-Scott rectangular pulses model is widely applied, both in its univariate and multivariate form. The estimation of the model parameters is traditionally carried out by using the method of moments, due to the difficulties involved in the application of maximum likelihood approaches. Recently, an approximated maximum likelihood estimator for univariate cluster models has been proposed. It makes use of the Whittle's approximation of the Gaussian maximum likelihood function. This approach, which provides consistent and normally distributed estimates, has been shown to satisfactorily perform when applied to some rainfall data observed in Great Britain. We propose here an extension of this method for estimating seasonal Neyman-Scott models, whose parameters are allowed to vary with the season. The seasonal spectral density of the data is estimated by taking the Fourier Transform of the correspondent seasonal sample autocorrelation function. The procedure has been tested by fitting synthetic hourly rainfall data.

1 Introduction

Precipitation modelling is a central topic in stochastic hydrology. Annual and monthly precipitation depths can be described by using autoregressive moving average (ARMA) models. However, daily precipitation occurrences and amounts are quite difficult to model. The main difficulties stem from the intermittent property of precipitation. It is even more difficult to model the precipitation series at a finer temporal resolution, such as hours or minutes.

In the latter years, point processes were widely used for the development of physically realistic intermittent rainfall models (see, for instance, Kavvas and Delleur, 1981; Waymire and Gupta, 1981; Smith and Karr, 1985; Rodriguez-Iturbe, 1986; Entekhabi *et al*., 1989). Rodriguez-Iturbe *et al.* (1987, 1988) have shown that the cluster-based rectangular pulses models, and in particular Neyman-Scott models and Bartlett-Lewis models (see Cox and Isham, 1980, for a review) are capable of adequately represent the rainfall process over a wide range of temporal scales of aggregation. In these clustered models, the occurrences of storms origins are assumed to follow a Poisson process. A random number of cells is associated with each storm event. Natural candidates for the distribution of the number of cells are the geometric distribution and the Poisson distribution. Precipitation of each cell is represented by a rectangular pulse whose intensity and duration are assumed to be exponentially distributed. In the Neyman-Scott model the position of these cells is determined by a set of independent and identically distributed random variables, which define the position of the origins of the cells with respect to the storm origin. In the Bartlett-Lewis process the intervals between successive cells are independent and identically distributed. Usually these latter random variables, which specify the location of the origins of the single cells, are assumed to be exponentially distributed in both models.

In the latter years these models have been widely considered by hydrologists for modelling the precipitation process. The Neyman-Scott model has been much more frequently applied and many modifications have been recently proposed in order to overcome some of its limitations (Cowpertwait, 1991, 1995).

The Neyman-Scott model today represents a powerful tool in many hydrological studies. It is typically used in Monte Carlo simulations, for generating records that preserve certain properties of the observed precipitation series, thus obtaining rainfall data which are different, but which are equally likely, with respect to the observed ones.

These data can be used, for instance, for recognising the hydrological effects produced by the inherent variability of the climate. To this end, it is often desirable to assess hydrological scenarios for a number of weather sequences different from the observed one but equally likely. Moreover, generation of synthetic data can be useful in order to extend the sample size of the historical record available, thus allowing to better inspect the statistical properties of the extremes. Synthetic rainfall data are also often used as input to hydrological models, in order to obtain synthetic river flows data. By varying the parameters of the rainfall and rainfall-runoff models accordingly one can retrieve indications about the effects on the river flows of changes in the climate or in the catchment characteristics, induced for instance by land-use change or anthropisation in general (e.g. Brath and Montanari, 1999).

However, a major limitation to the use of the Neyman-Scott model stems in the lack of suitable parameter estimation schemes. Maximum likelihood estimators proposed in the past demand an extensive computational effort and generally are not suitable for practical application (Smith and Karr, 1985; Foufoula-Georgiou and Guttorp, 1986). Thus, parameter estimation was so far mainly performed using the method of moments. Accordingly, estimates of various combinations of first- and second-order statistics from historical precipitation time series are equated to their theoretical expressions, which are function of the model parameters. Least squares techniques are usually employed to minimise the differences between theoretical and computed values of the above statistics. This approach suffers from the disadvantage that the parameter estimates can vary greatly depending on the statistics which are used in the fitting procedure.

Recently, these drawbacks were partially overcome by Chandler (1997), who proposed an approximate maximum likelihood technique for estimating the parameters of a wide class of point-process based rainfall models. His approach is based on the application of the Whittle's (1953) approximation of the Gaussian maximum likelihood function and provides asymptotically consistent and normally distributed estimates for both Gaussian and non-Gaussian data. The parameter estimation is performed by numerically minimising an appropriate objective function, which is obtained by comparing the spectral density of the model and the periodogram of the data. However, such objective function is often characterised by the presence of many local minima. This is a common occurrence when dealing with point processes. In fact, widely separate regions of the parameter space can often give raise to similar objective function values, even when using other methods of parameter estimation (Onof and Weather, 1993). Therefore, Chandler (1997) proposed to run the minimisation algorithm with different starting values of the parameters, in order to increase the probability to find the absolute minimum of the objective function.

In this paper, Chandler's (1997) approach is used in order to estimate the parameters of seasonal models. In order to account for seasonality, the year is thus divided into periods (seasons), and maximum likelihood estimation is applied to each period. Homogeneity of the process in each season is assumed. Moreover, a powerful genetic algorithm, namely the SCE-UA routine proposed by Duan *et al*. (1992), is applied here in order to minimise the objective function. This approach performs the estimation by exploring the whole parameters space, thus providing a more efficient minimisation and avoiding to run the minimisation routine with different starting parameter values.

The procedure described here can be applied to a wide class of point processes. However, we will focus on the Neyman-Scott model, which is the most widely applied by the hydrological community.

The proposed estimation method has been tested by fitting some synthetic rainfall series. The derived estimates have been compared with the analogous ones obtained by applying the method of moments.

The next section of the paper describes the Neyman-Scott rectangular pulses model. The third section is devoted to the description of the maximum likelihood approach considered by Chandler (1997), while the fourth section describes the extension to this approach to the case of seasonal models. The fifth section describes the application of the estimators and the comparison with the method of moments. The last section reports some concluding remarks.

2 OUTLINE OF THE NEYMAN-SCOTT MODEL

The Neyman-Scott rectangular pulses rainfall model introduced by Rodriguez-Iturbe *et al.* (1987) is a particular form of stationary and clustered point process. Storm origins (shown in Figure 1 as black squares) occur according to a Poisson process with arrival rate λ, so that the time between adjacent storm origins is an exponential random variable with mean $1/\lambda$. Each storm origin generates a random number C ($C = 1, 2, 3, \ldots$) of rain cells. The number of rain cells associated with a certain storm is usually assumed to be a Poisson random variable with mean ν. The waiting time, after a storm origin, for the starting of a single rain cell (shown in Figure 1 as white circles) is an exponential random variable with parameter β. Each rain cell has a random duration and a random intensity, where the intensity is held constant throughout the cell duration. Both the intensity and the duration are assumed to be exponentially distributed with parameter x and η respectively. The total rainfall intensity at time t is the sum of the intensities of all active rain cells at that time. Figure 1 reports a sketch of the schematisation of the hyetograph operated by the model.

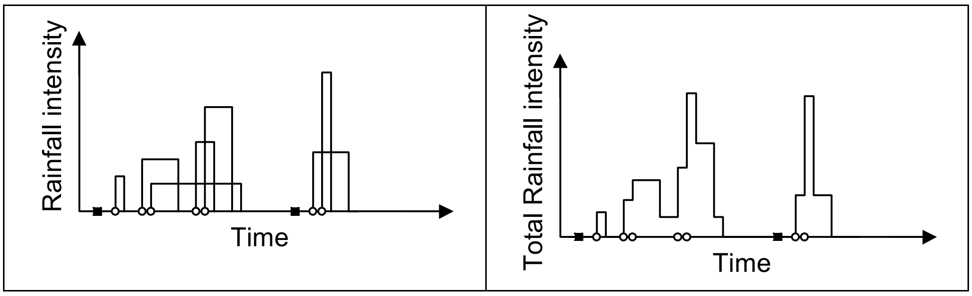

Figure 1 – Schematisation of the hyetograph operated by the Neyman-Scott model. Storm origins and cell origins are marked with black squares and white circles respectively.

The Neyman-Scott model provides a continuous-time representation of the rainfall process. The resulting rainfall depths are often aggregated over assigned time steps. The model can be used for modelling rainfall data on a wide range of time scales, provided the time step between successive observations is fine enough in order the intermittence of the process to be preserved.

The model structure is characterised by five parameters which are to be estimated by fitting historical rainfall data. As previously mentioned, model estimation was usually performed in the past by using the method of moments, which consists in equating estimates of various combinations of first- and second-order statistics from historical precipitation time series to their theoretical expressions, which are dependent on the model parameters. Least squares techniques are usually employed to minimise the differences between theoretical and computed values of the above statistics.

In order to account for seasonality, the model parameters are usually estimated on a seasonal basis. Accordingly, the year is divided into periods (seasons) and stationarity of the process is assumed in each of them. The estimation procedure is then performed for each period and thus the model parameters can assume different values in different seasons.

Further details on the Neyman-Scott model can be found in Rodriguez-Iturbe *et al.* (1987) and Burlando (1989).

3 Maximum likelihood estimation for the Neyman-Scott rectangular pulses model

We describe here the theoretical basis for an approximate maximum likelihood estimation procedure for the Neyman-Scott model. The interested reader is invited to refer to Chandler (1997) for more details.

Let us denote with X_t a stochastic process of mean rainfall intensities over a time step Δ, at an assigned raingauge. The process X_t is assumed to be strictly stationary and not affected by long-memory or long-range dependence. This implies that the scale of fluctuation of the process is finite (Mesa and Poveda, 1993) and that the second-order spectral density of the process exists and is finite.

Let us now suppose that we are interested in estimating the parameters of a model, whose parameter vector is $\Theta = (\theta_1, \theta_2, \ldots, \theta_k)$. When the sample size N of the available time series is odd, and all frequencies except zero are included in the analysis, the Whittle's approximate maximum likelihood estimation is carried out by minimising

$$Q(\Theta) = \sum_{j=1}^{m} \left[\frac{I(\omega_j)}{4\pi h_X(\omega_j, \Theta)} + ln\, h_X(\omega_j, \Theta) \right]. \qquad (1)$$

The argument is essentially the same when N is even and when zero frequency is included, although the expression involved is more complicated. Here $h_X(\omega_j,\Theta)$ is the spectral density of the model at the frequency ω_j, which depends on the model parameters, $I(\omega_j)$ is the periodogram of the data and m is the integer part of $(N - 1)/2$. Chandler (1997) shows that the estimation procedure obtained by minizing (1) can be regarded as a quasi-likelihood technique. It can be proved that, under mild conditions, the Whittle's estimator is consistent and normally distributed. The variance of the parameter estimates depends on the fourth order spectrum of the rainfall intensity process. However, computation of the confidence limits of the estimated parametes is not very useful when dealing with many of the models currently in use. Apart from the difficulty in obtaining the fourth-order spectrum, there are often problems because widely separated regions of the parameter space can lead to similar values of the objective function. This kind of problem arises also when dealing with other methods of parameter estimation (Rodriguez-Iturbe *et al.*, 1988; Onof and Weather, 1993).

The application of the Whittle's method has been extensively discussed in the literature, even for the case of non-Gaussian data affected by long-range dependence (see, for instance, Beran, 1994; Samorodnitsky and Taqqu, 1994). Further details about the computation of the spectral density of the model, the periodogram of the data and about the numerical minimisation of (1) are given here below.

3.1 Computation of the spectral density of the model

The spectral density of a generic parametric discrete time process, whose parameter vector is Θ, is defined by

$$h_X(\omega_j,\Theta)=\frac{1}{2\pi}\sum_{r=-\infty}^{\infty}c_r(\Theta)e^{-i\omega_j r\Delta} \quad \omega_j \in (-\pi/\Delta,\ \pi/\Delta), \tag{2}$$

where $c_r(\Theta)$ is the autocovariance function of the process at lag r. The above spectral density can also by expressed by (Chandler, 1997)

$$h_X(\omega_j,\Theta)=\sum_{k=-\infty}^{\infty}\left\{g\left[\omega_j+\frac{2k\pi}{\Delta},\Theta\right]\cdot\left[\frac{sin((\omega_j\Delta+2k\pi)/2)}{(\omega_j\Delta+2k\pi)/2}\right]^2\right\} \quad |\omega_j| \le \pi/\Delta\ , \tag{3}$$

where $g(\omega_j,\Theta)$ is the theoretical spectral density of the underlying continuos time process. This result derives from the fact that the discretisation in time steps of length Δ is equivalent to first applying a uniform filter of width Δ to the original process, then sampling the filtered process at intervals of length Δ.

The form of the spectral density $g(\omega_j,\Theta)$ is known for a wide class of stochastic rainfall models based on point processes. For the Neyman-Scott model, described in Section 2, it can be written as

$$g(\omega_j,\Theta)=\frac{1}{2\pi\omega_j^2}\left\{2\pi g_N(\omega_j)x\left|1-\phi(\omega_j,\eta)\right|^2+2\lambda\left(x^2+\sigma^2\right)\left[1-\Re\left(\phi(\omega_j,\eta)\right)\right]\right\} \quad |\omega_j| \neq 0, \qquad (4)$$

where η and σ are respectively the mean and the variance of the cell intensity distribution (which is exponential in the case of the Neyman-Scott process), $\phi(\omega_j,\eta)$ is the characteristic function of the cell duration distribution, $\Re(z)$ indicates the real part of the complex number z and $g_N(\omega_j)$ is the theoretical spectral density of the so-called *driving-process*, which is composed by the storm origin, the rain cells number and the cell starting time processes. When $\omega_j = 0$ it is necessary to apply a limiting operation to obtain the result.

For the Neyman-Scott process, $g_N(\omega_j)$ is given by

$$g_N(\omega_j)=\frac{\lambda}{2\pi}E[C(C-1)]\left|\phi(\omega_j,\beta)\right|^2 \quad) \omega_j, \qquad (5)$$

where $\phi(\omega_j,\beta)$ is the characteristic function of the probability distribution of the waiting time, after a storm origin, for the starting time of the rain cells, $E(\alpha)$ is the expected value of the random variable α and C is the number of rain cells associated to each storm origin defined above.

3.2 Computation of the periodogram of the data

The spectral density $h_X(\omega_j,\Theta)$ can be estimated by computing the periodogram of an observed time series which is a realisation of the investigated stochastic process X_t. The periodogram is defined as

$$I(\omega_j)=\frac{N}{2}\left(A_{\omega_j}^2+B_{\omega_j}^2\right) \qquad 0 \;[\; \omega_j \;[\; 0.5 \qquad , \qquad (6)$$

where A_{ω_j} and B_{ω_j} are the sample Fourier coefficients for the frequency ω_j.. When N is odd, they can be computed by

$$A_{\omega_j}=\frac{2}{N}\sum_{t=1}^{N}X_t\,cos\left(\omega_j t\Delta\right) \qquad (7)$$

$$B_{\omega_j} = \frac{2}{N}\sum_{t=1}^{N} X_t sin(\omega_j t\Delta) \tag{8}$$

where $\omega_j = 2\pi j/N\Delta$, j = 1, 2, …. (N - 1)/2. When N is even, (7) and (8) apply for j = 1, 2, …. ($N/2$ - 1) but

$$A_{\omega_{N/2}} = \frac{1}{N}\sum_{t=1}^{N}(-1)^t X_t \quad , \tag{9}$$

$$B_{\omega_{N/2}} = 0 \quad . \tag{10}$$

Note that the highest frequency is 0.5 cycles per time interval because the smallest period is 2 intervals.

The periodogram can be also computed by taking the Fourier cosine transform of the autocovariance function of the data, that is,

$$I(\omega_j) = 2\left\{c_0 + 2\sum_{k=1}^{M} c_k \cos 2\pi\omega_j\right\} \tag{11}$$

where M is the number of the autocovariance coefficients c_k, k = 1, 2, …., M, significantly different from zero. For details about the practical choice of M see Kottegoda (1980).

3.3 Numerical minimisation of the objective function

The objective function given by (1) was minimised here by applying a genetic algorithm, namely the SCE-UA proposed by Duan *et al*. (1992), which was found to be the most effective among several global optimisation methods.

4 APPLICATION OF THE ESTIMATOR TO SEASONAL DATA

We mentioned in Section 3 that, when modelling rainfall data using point processes, seasonality of the rainfall process is usually accounted for by estimating the model parameters on a seasonal basis. Accordingly, the year is divided into periods (seasons) and stationarity of the process is assumed in each of them. The estimation procedure is then performed for each period and thus the model parameters can assume different values in different seasons.

Application of the Whittle's estimator to seasonal data requires the computation of the periodogram for each season. This is complicated by the fact that the observed rainfall data are available in form of multiple realisations, one for each year, of the rainfall process related to the assigned season.

Two approaches are possible for computing the periodogram in this case. The first is to estimate the periodogram referred to the assigned season by averaging the yearly periodograms referred to the same season, which can be computed separately for each year of the observation period. When dealing with limited length seasons, this approach presents the weakness that the periodogram can be estimated only for a limited range of Fourier frequencies. The second possible approach, which was used here, is to compute the seasonal periodogram using (11). The seasonal autocovariance coefficient at lag i, referred to the whole data observed in the season k, which will be denoted here below as $c'_{i,k}$, can be estimated by computing the averages of the corresponding values estimated for each year, that is,

$$c'_{i,k} = \frac{1}{L}\sum_{p=1}^{L} c''_{i,k,p} , \tag{12}$$

where L is the number of years of data and $c''_{i,k,p}$ denotes the seasonal autocovariance coefficient at lag i, referred to the season k and year p.

The choice of the value of M is quite important (see equation (11)). When dealing with hydrological time series, usually the large scale characteristics of the periodogram ordinates are provided by the low-lag autocovariance coefficients, while the high-lag ones provide more details about the small scale fluctuations. Thus, reducing M leads to obtain smoother periodograms. Therefore, on one hand, one would naturally tend to choose high Ms in order to obtain a better detail in the computation of the periodogram. On the other hand it should be noted that the periodogram is an inconsistent estimator of the spectral density and it needs to be smoothed in order to be useful (Priestley, 1981). Moreover M, which in principle could be season dependent, cannot exceed the length of the corresponding season and the higher values of M are computed on a lower number of data and therefore are less reliable. Finally, it should be noted that, when dealing with rainfall data, the number of the significant autocovariance coefficients is usually small and therefore a good approximation in the periodogram computation can be obtained also using values of M which are relatively limited.

Accordingly to the previous discussion, M was chosen here equal to 10. Higher values were found not to improve significantly the results when referring to the examined synthetic rainfall series.

5 TESTING THE ESTIMATOR PERFORMANCES ON SYNTHETIC RAINFALL DATA

The maximum likelihood estimator described above was tested by analysing synthetic rainfall series, generated using a Neyman-Scott model. 50 years of synthetic hourly rainfall depths were generated by using the parameter values reported in Table 1.

Neyman-Scott parameter estimation has been performed using the method of moments and the maximum likelihood estimator described above. In this latter case, the optimisation procedure has been started at a point in the parameter space different from the one representative of the true set of parameter values. The resulting parameter estimates have been compared with the respective true values, in order to assess the reliability of the Whittle's approach with respect to the traditional method of moments. The estimation procedure has been carried out by subdividing the year in 12 season, corresponding to the 12 calendar months. Therefore, the estimation has been carried out on seasonal basis, even if the data were generated by assuming the same parameter values in all the seasons.

The method of moments was applied by optimising the fit of the mean, the variance, the proportion of dry days and the lag-one correlation of the hourly data in each season.

When implementing the maximum likelihood approach, the infinite summation in (3) was limited to 20 terms, for $k = \pm 1, \pm 2, \ldots, \pm 10$, since the spectral density becomes negligible at high frequencies.

Neyman-Scott parameter	λ $(1/h)$	ν	β $(1/h)$	x (h/mm)	η $(1/h)$
Parameter values	0.01	5	0.3	0.2	3

Table 1. Values of the parameter of the Neyman-Scott model used for the generation of the synthetic rainfall data. The dimension of each parameter is reported in parenthesis (the symbol *h* denotes hours)

The results of the estimation are reported in Table 2, which shows the mean (over the 12 seasons) absolute relative errors of estimation *Mre* for each parameter. In Figure 2 is reported the index Re, which allows to compare the performances of the two estimation methods, which is given by

$$\mathrm{Re} = \frac{Mre_m}{Mre_l}, \tag{12}$$

where Mre_m and Mre_l are the mean relative errors of estimation for each Neyman-Scott parameter when using the method of moments and maximum likelihood respectively

Neyman-Scott parameter	λ	β	ν	x	η
Mre_m	0.005	1.53	3.87	0.13	2.15
Mre_l	0.018	2.98	1.62	0.07	0.61

Table 2. Mean (over the 12 seasons) absolute percentage errors Mre_m and Mre_l of estimation for the Neyman-Scott parameters, by using the method of moments and maximum likelihood respectively.

The results highlight a satisfactory reliability of the Whittle's method when compared with the traditional method of parameter estimation. In particular, maximum likelihood resulted much more precise in the estimation of the parameters ν, x and η, while the method of moments performed better in the estimation of λ and β. It can be observed that some of the resulting relative errors are quite high for both the estimation methods. This is a quite common occurrence when estimating the parameters of point process models. In fact, it was mentioned in Section 3 that often widely separated regions of the parameter space lead to similar values of the objective function and similar values of the main statistics of the data (Onof and Weather, 1993).

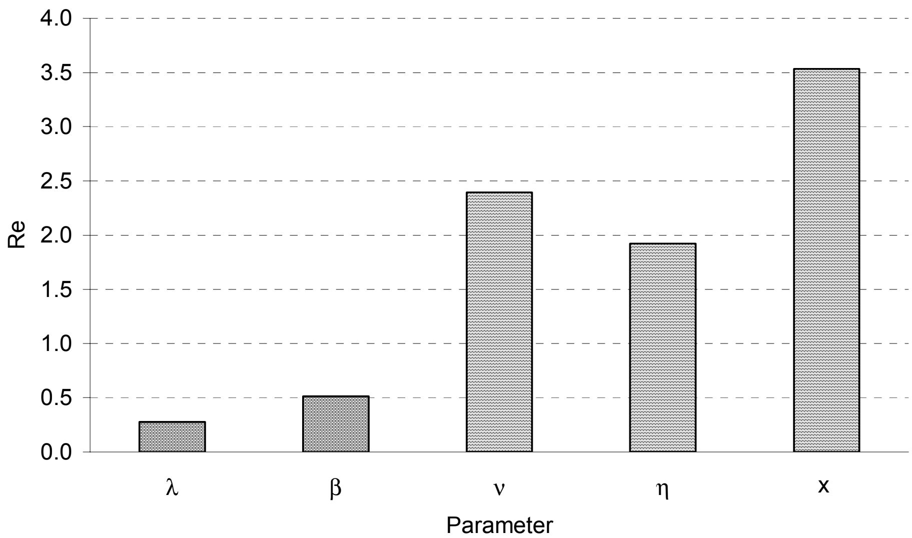

Figure 2. Comparison between the method of moments and maximum likelihood. Ratio Re between the mean relative errors found when estimating each parameter (see equation (12)). Re > 1 is obtained when maximum likelihood outperforms the method of moments.

It is interesting to analyse how maximum likelihood performs in fitting the rainfall data statistics which were selected for estimating the model parameters using the method of moments. Since these specific statistics, which in the present case are the mean, the proportion of dry hours, the variance and the lag-one correlation, are surely best fitted by the method of moments, the performances of maximum likelihood in fitting them are a useful benchmark, which allows to assess the fitting capability of this latter estimation method with respect to the best ones of the method of moments. The obtained mean (over the 12 seasons) absolute relative errors are reported in Table 3.

As it was expected, the method of moments performs better in reproducing all the considered statistics but the mean, which is astonishingly better reproduced by maximum likelihood. The performances of this latter method in reproducing the proportion of dry hours were found satisfactory, while the lag-one correlation and the variance resulted not well reproduced. It is well known that to fit the correlation of rainfall data aggregated at fine time span using point processes is not an easy task. Thus, the error observed in reproducing the variance appears to denote a more severe limitation of maximum likelihood estimation than the one observed in the lag-one correlation coefficient. The lack of efficiency of maximum likelihood in this respect is currently under investigation by analysing longer synthetic rainfall series.

Statistics	μ	P_d	σ^2	ρ_1
Mre_m	0.6	0.1	8.0	7.7
Mre_l	0.3	1.3	27.3	14.8

Table 3. Mean (over the 12 seasons) absolute relative errors Mre_m and Mre_l in fitting some first and second-order statistics of the synthetic rainfall data using the method of moments and maximum likelihood respectively.

7 CONCLUSIONS

In this paper, an approximate maximum likelihood estimator for seasonal rainfall models based on point processes has been considered, based on the approach recently proposed by Chandler (1997). Model estimation is performed by applying the Whittle's approximation to the Gaussian maximum likelihood function in the spectral domain, which provides asymptotically consistent and normally distributed estimates. Seasonality is accounted for by dividing the year in period (seasons) and performing the parameter estimation separately for each of them. Stationarity of the process in each period is assumed.

The estimator objective function is minimised here numerically by applying a genetic algorithm, which allows to explore the whole parameter space for the search of the absolute minimum.

The estimator was tested by fitting 50 years of synthetic hourly rainfall data, generated using a Neyman-Scott model with assigned parameter values. The Whittle's estimator was found to provide quite satisfactory performances. However, some lack of accuracy was found in fitting the variance of the data. This aspect of the analysis is the subject of on-going work.

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